ASSESSMENT ON SPARE PARTS REQUIREMENT BASED ON RELIABILITY'S CHARACTERISTICS

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ABSTRACT: The purpose of this study is to estimate the spare parts necessary, for a determined period of time of operation for a machine or installation, based on reliability studies performed on their components, which allows a proper sizing of the stock, with beneficial effects on ensuring the continuity of their operation, especially on the economic effects.

Keywords: spare parts, reliability, estimation

1. GENERAL CONSIDERATIONS ON THE ESTIMATION OF THE NECESSARY SPARE PARTS

In the specialty literature there is a great number of references in the field of spare parts supply, particularly in their logistics.

In general, most of these works refer to repairable systems and spare parts management. They are mainly using the expectancy theory to determine the stock of spare parts in order to ensure an availability requirement to the system.

On the other hand, the quantitative methods based on the theory of reliability allow the estimation of the intensity or the rate of failure for the supplied (purchased) and/or stored items, which is used to determine more accurate application rates.

Ensuring the systems availability implies that spare parts are always available on request. However, estimating and calculating the required number of spare parts required in order to ensure the requested availability, taking into account the technical and economic requirements (reliability, maintainability, lifespan cost etc..) were

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considered and studied very few times.

In extremely few cases, the specialty literature on the calculation of spare parts necessary based on the reliability characteristics of a product has considered operating conditions as a factor that influences the reliability. The estimates carried out are not precise enough, because in the actual operation situation, it was shown that there are many factors, other than time, which have a considerable influence on the reliability characteristics of components and systems.

Environmental conditions in which an equipment operates, such as temperature, humidity, existence of dust, etc., often have a considerable influence on the reliability characteristics of the product, conditions that must be considered when designing the logistic support and adopting the maintenance strategy.

2. MODEL OF FAILURE INTENSITY FOR ESTIMATING THE SPARE PARTS NECESSITY

This model for determining the amount of spare parts stock is based on the failure intensity of the product that is considered a replacement part, which can be determined, with a certain confidence level, only through a study of reliability.

At the moment $t=0$ is set in work the first spare parts, it fails at the moment $t=\tau_1$ and it is replaced (in a negligible period of time) by a second spare part, which breaks at the moment $t=\tau_1+\tau_2$ and it's being replaced by the third spare part.

The question arises when determining, with a given probability, γ, or a given level of confidence, the number n of spare parts that must be kept in reserve so that all the necessities can be covered for a cumulative operating time T.

To answer this question it is necessary to determine the smallest integer value of n for which the inequality is satisfied,

$$\Pr\left\{\tau_1 + \tau_2 + \ldots + \tau_n > T\right\} \geq \gamma . \quad (1)$$

In general, it is assumed that τ_i are random values, independent and positive, with the same distribution function $F(x)$ and the same probability density $f(x)$.

Also, the average value of variables is $E[\tau_i]=E[\tau]=MTTF$, i.e. the average time until failure, and variation or dispersion, $D=Var[\tau_i]=Var[\tau]$.

If the number n of spare parts is calculated with

$$n = \frac{T}{MTTF},\ pcs \quad (2)$$

can not cover the needs, for a longer period of time T, only with a probability of 50%.

Therefore, it is necessary to reserve more than $T/MTTF$ spare parts, so it would be able to achieve a probability higher than 0,5.

In (1), the left side can be expressed, using the distribution function $F(x)$, through the relation,

$$\Pr\{\tau_1+\tau_2+\ldots+\tau_n>T\}=1-F_n(T), \tag{3}$$

where:

$$\begin{aligned}&F_1(T)=F(T);\\&F_n(T)=\int_0^T F_{n-1}(T-x)f(x)\,dx,\ \ n>1.\end{aligned} \tag{4}$$

Exponential, gamma, normal distribution laws, mainly used in reliability theory are expressed by simple relations, which allows to directly solve the integrals, resulting Poisson, gamma and normal distributions. In the case of an exponential distribution, a case encountered quite frequently in reliability practice,

$$\Pr\left\{\sum_{i=1}^{n}\tau_i>T\right\}=\sum_{i=0}^{n-1}\frac{(\lambda T)^i}{i!}e^{-\lambda T}\ , \tag{5}$$

where λ represents the failure intensity in *fails/hour*, that becomes the request intensity of the spare part and which has a constant value.

For biparametric Weibull distribution, with distribution function $F(x)=1-\exp\left[-(\lambda\, x)^{\beta}\right]$, or standardized form, $F(x)=1-\exp\left[-(x/\eta\,)^{\beta}\right]$, which is the most widely distribution law used in mechanical field, the application must be solved numerically. In Figure 1 are plotted the solutions to this problem, depending on the shape parameter, β, the biparametric Weibull distribution law and the imposed γ probability. The dashed lines in the diagrams from Figure 1 are the results obtained using the central limit theorem, as approximate graphics. In these graphical representations, the required number of spare parts, for the four levels of confidence, is being shown by the ratio of the period considered and the average time until failure, *MTTF*, value that is calculated with calculation relations specific to each distribution law that is governing the process of analyzed product failure.

For larger values of n, applying the central limit theorem, we can get an approximate solution for a broad class of distribution functions $F(x)$. Thus, for $Var[\tau]<\infty$,

$$\lim_{n\to\infty}\Pr\left\{\frac{\sum_{i=1}^{n}(\tau_i-E[\tau])}{\sqrt{n\,Var[\tau]}}>x\right\}=\frac{1}{\sqrt{2\pi}}\int_x^{\infty}e^{-\frac{y^2}{2}}dy=1-\Phi(x)=\Phi(-x), \tag{6}$$

where $\Phi(x)$ is Laplace function.

By applying the transformation $\sqrt{n\ Var[\tau]}+n\ E[\tau]=T$ it's obtained

$$\lim_{n\to\infty} \Pr\left\{\sum_{i=1}^{n} \tau_i > T\right\} = \frac{1}{\sqrt{2\pi}} \int_{\frac{T-nE[\tau]}{\sqrt{n\,Var[\tau]}}}^{\infty} e^{-\frac{y^2}{2}}\,dy = \gamma\,. \tag{7}$$

For given values of the parameters γ, T, $E[\tau]$ and $Var[\tau]$, the value of n is determined by calculation or using the normal standard distribution tables.

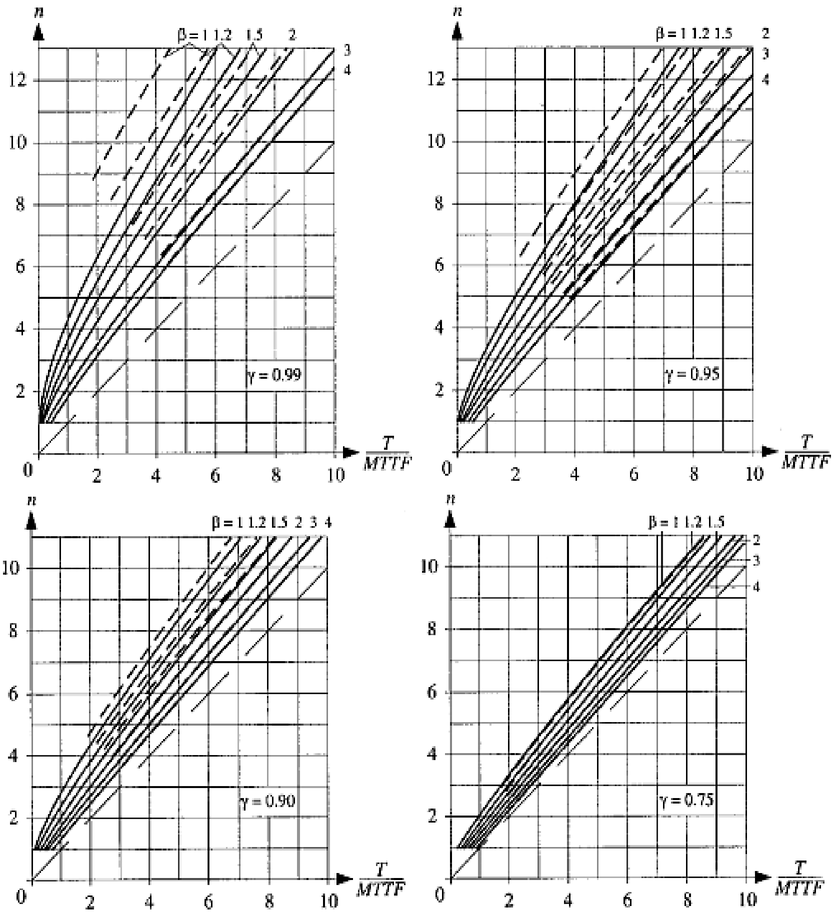

Fig. 1. Number of spare parts necessary for covering the cumulative uptime according to bi-parametric Weibull distribution using the failure intensity model

By introducing the notation

$$\frac{T - n\,E[\tau]}{\sqrt{n\,Var[\tau]}} = -d\,, \tag{8}$$

results,

$$n = \left[\frac{d\,(CV)}{2} + \sqrt{\left(\frac{d\,(CV)}{2} \right)^2 + \frac{T}{E[\tau]}} \right]^2 , \tag{9}$$

where *CV* represents the variation of the distribution factor, which is calculated by the relation,

$$CV = \frac{\sqrt{Var[\tau]}}{E[\tau]} = \frac{\sqrt{D}}{E[\tau]} = \frac{\sigma}{E[\tau]} , \tag{10}$$

where *D* represents the dispersion and σ average standard deviation of the random variable.

Table 3 presents the values of the parameter *d*, for usual values of the confidence level γ, obtained by using the inverse Laplace function, $\Phi^{-1}(\gamma)$.

Table 1. **Values of the parameter d for different confidence levels γ**

γ	0,99	0,95	0,90	0,75	0,50
$d = \Phi^{-1}(\gamma)$.	2,33	1,64	1,28	0,67	0,00

The relations (8), (9) and (10) are valid for any distribution functions *F*(*x*) used in the study of reliability. For standardized biparametric Weibull distribution, the variation factor, CV, is calculated with

$$CV = \frac{\sqrt{D}}{m} = \frac{\sqrt{\Gamma\left(1+\frac{2}{\beta}\right) - \left[\Gamma\left(1+\frac{1}{\beta}\right)\right]^2}}{\Gamma\left(\frac{1}{\beta}+1\right)} \tag{11}$$

and it does depend only by the shape parameter β. The variation of this factor is being shown in Figure 2.

For when the biparametric Weibull distribution law is governing the operation process, respectively failure, of the product studied, determining the number of spare parts for a period of time, results from the graphical representations shown in Figures 3…7, for different confidence levels and different shape parameters of the distribution.

In these graphical representations, shape parameter values of biparametric Weibull distribution will be considered from top to bottom in ascending order, the values being those recorded in the field of representation.

Comparing the diagrams in Figure 1, with the ones in the representations 3…7, in regard to determine the required number of spare parts, in the exact version, respectively the estimated one, it is observed that there are differences, but not essential, especially that following calculation is adopted the smallest integer value.

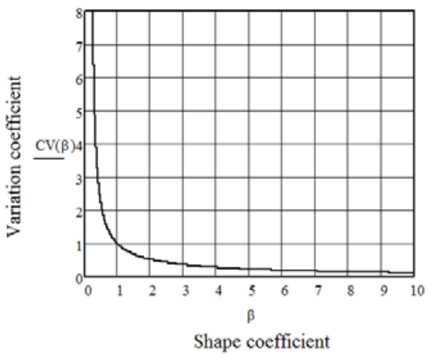

Fig. 2. Variation of variation factor for biparametric Weibull distribution

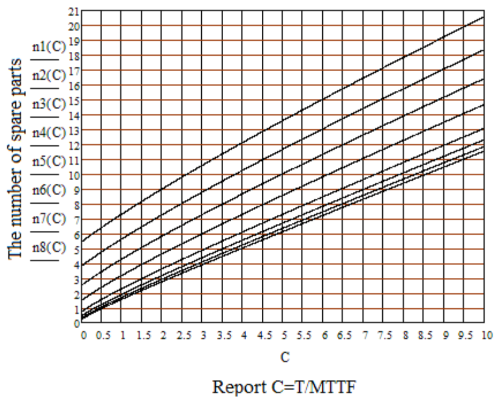

Fig. 3. Number of spare parts related to the average time of failure, biparametric Weibull distribution with shape parameter 1; 1,2; 1,5; 2; 3; 4, 5; 6, confidence level 0,99, intensity of the fault model

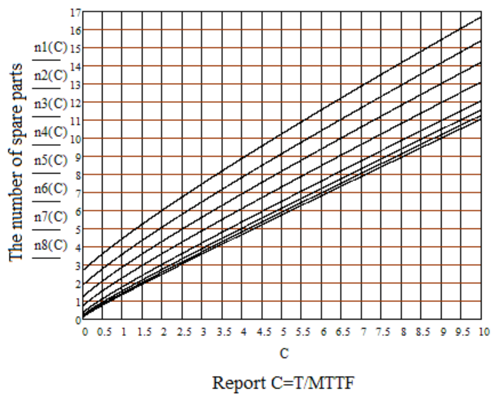

Fig. 4. Number of spare parts related to the average time of failure, biparametric Weibull distribution with shape parameter 1; 1,2; 1,5; 2; 3; 4; 5; 6, confidence level 0,95, intensity of the fault model

For any of the distribution laws that are governing the operation or failure process of a product, knowing their parameters, from the equation 11 can be represented the relationship $n=f(T)$, from which immediately results the number of spare parts needed, depending on the cumulative operating time.

In the cases mentioned above it was assumed that spare parts are not repairable. The case of repairable spare parts are treated with the theory of stochastic processes. One of the specimens often encountered is that of repairable passive redundancy 1 of n or k of n, where the spare part is used k^{th} time in that unit.

Given that the supply is centralized through supply bases, the determination of the necessity after the models from reliability theory leads to a decrease of spare parts stocked and thereby of acquisition costs.

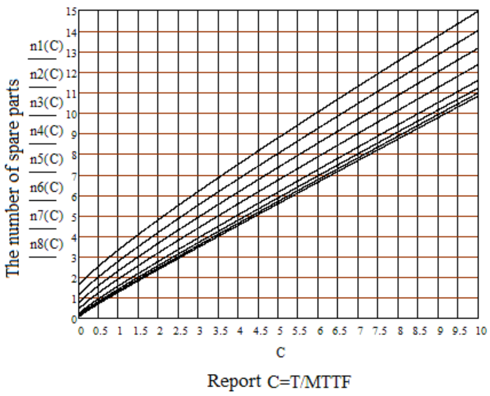

Fig. 5. Number of spare parts related to the average time of failure, biparametric Weibull distribution with shape parameter 1; 1,2; 1,5; 2; 3; 4; 5, 6, confidence level 0,90, intensity of the fault model

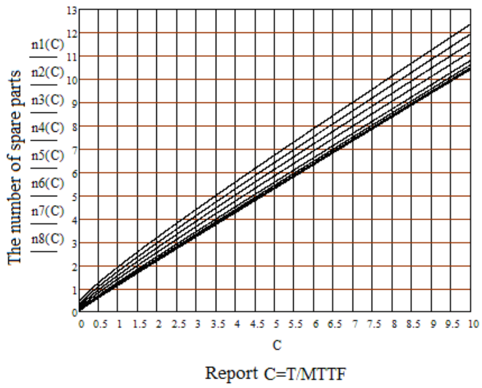

Fig. 6. Number of spare parts related to the average time of failure, biparametric Weibull distribution with shape parameter 1; 1,2; 1,5; 2; 3; 4, confidence level 0,75, intensity of the fault model

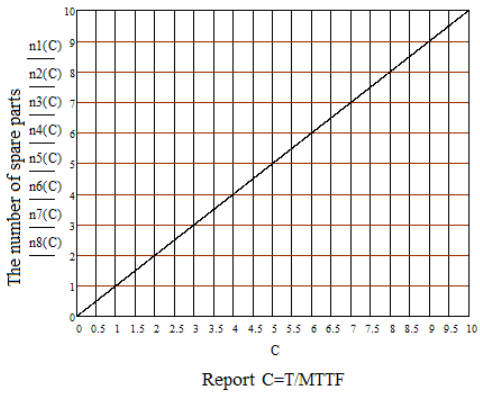

Fig. 7. Number of spare parts related to the average time of failure, biparametric Weibull distribution with shape parameter 1; 1,2; 1,5; 2; 3; 4 confidence level 0,50, intensity of the fault model

3. CASE STUDY

In this case study, there's the question to determine the number of spare parts needed for the maintenance of loading, transporting and storing machines analyzed, in terms of providing the necessary of linear hydraulic engines for bucket handling, the piston pumps from the braking hydraulic circuit, as well as brake pads within the braking system.

The input data necessary for determining the number of spare parts, as in theoretical explanations stated above, is presented in Table 4. The parameters considered are the ones specific to Weibull distribution, considered as the one that best characterizes the functionality of the analyzed mechanical elements. The considered time period of 10000 hours corresponds to about four years of operation.

Table 5 presents the values for the number of spare parts required for a period of four years, estimated using specific charts packages specific to the exact and the approximate method, figures 1 and 3... 7.

In Figures 8, 9 and 10 are given the necessary spare parts according to the time of operation for the three components of the loading, transporting and storing machine under study. The graphical representations are obtained using the relationship 9, and the parameters included in Table 4. The confidence levels are placed on the graphical representation from top to bottom with the values from the field of representation.

Table 4. **Parameters required to determine the required number of spare parts**

Item no.	Analyzed product	Parameter			Operating time, T, h	Ratio T/MTTF
		β	η, hrs.	MTTF, hrs		
1	Linear hydraulic engine	2,065	2451	2171	10000	4,60
2	Hydraulic pump	5,005	7442	6834		1,46
3	Brake pads	6,543	1622	1512		6,61

Table 5. **Necessary spare parts for a period of four years of operation**

Item no.	Analyzed product, Spare part	Spare parts number, n								
		Exact method				Approximate method				
		Confidence level, γ								
		0,99	0,95	0,90	0,75	0,99	0,95	0,90	0,75	0,50
1	Linear hydraulic engine	7	6	5	5	8	6	6	5	4
2	Hydraulic pump	2	1	1	1	2	2	2	1	1
3	Brake pads	8	7	7	6	8	7	7	6	6

From the graphic representations presented results a very little difference, practically negligible, between the values obtained using the exact and approximate method, given the fact that the values of the required number of spare parts is rounded into the integer value by default.

Nomograms in Figures 1, 3 ... 7 have a general nature, being used for any of the products studied. They allow direct, immediate reading, depending on the considered operating time, on the necessary spare parts, once it's being known the average operating time until failure of the product being analyzed and the value of shape parameter β of the Weibull distribution.

Representations in Figures 8, 9 and 10 are more complete by considering the operating time, in *hours*, imposing the knowledge of β and η parameters of Weibull distribution specific to a particular type of product, with the help of which is calculated the quantities entering in the calculation relationship 9.

Also, graphic representations show a great need of spare parts, for all three products, while operating for four years, which demonstrates once again the low reliability of the analyzed products.

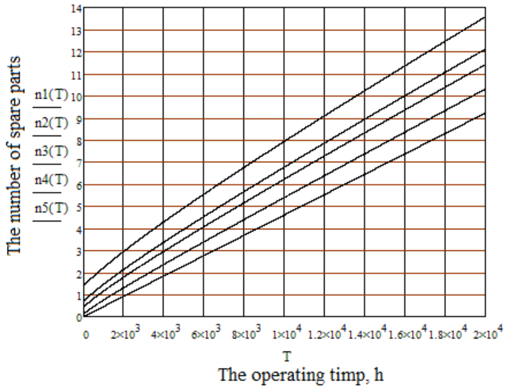

Fig. 8. The number of spare parts depending on the cumulative operating time for linear hydraulic engine, confidence levels 0,99; 0,95; 0,90; 0,75; 0,50, intensity of the fault model

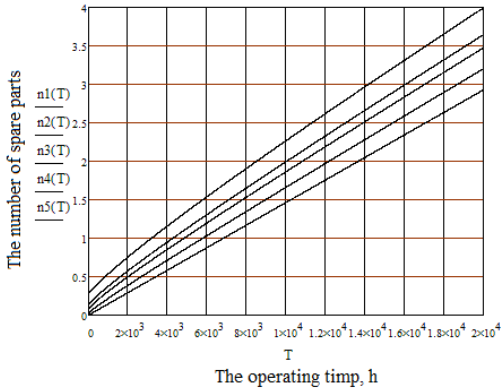

Fig. 9. The number of spare parts depending on the cumulative operating time for hydraulic pump from braking circuit, confidence levels 0,99; 0,95; 0,90; 0,75; 0,50, intensity of the fault model

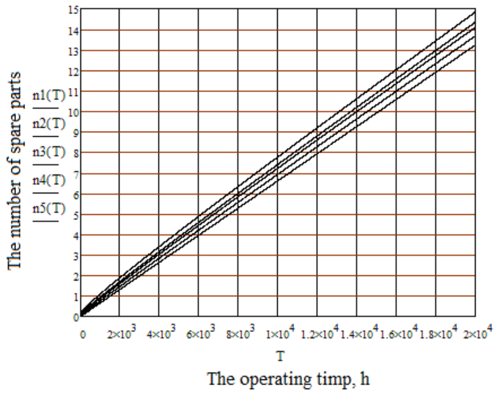

Fig. 10. The number of spare parts depending on the cumulative operating time for braking pads, confidence levels 0,99; 0,95; 0,90; 0,75; 0,50, intensity of the fault model

4. CONCLUSIONS

The estimating activity of the necessary spare parts is an integral part of maintenance based on reliability, which is a concept of using the feedback from the operation of facilities in predictive maintenance, based on reliability calculations and that aims at the optimization of maintenance strategies under given technical and economic data.

The failure intensity is a coherent model that is based on the possibility of calculating the failure intensity of the product, expressed as the number of failures that occur in an aggregate period of time, which in fact is nothing but the intensity of demand of the product, which is the spare part.

Theoretical considerations specific to the probability theory, combined with the technical aspects of the problem analyzed, led to a mathematical relationship, in which the required number of spare parts is expressed depending on the parameters that define the distribution law that is governing the operating and failure process of the product, including on the distribution of the necessary number of spare parts.

For the Weibull distribution law, which best characterizes the functionality of mechanical products, the calculation relationship was solved for different levels of confidence, resulting in a quasi-precise solution based on the application of numerical methods, and an estimate one as a consequence of applying the central limit theorem.

Both solutions were accomplished by developing two sets of nomograms that allow rapid calculation of the necessary spare parts for a preset time, for any products that validates the Weibullian behavior and for which is known the shape factor of the theoretical distribution and the average operating time until failure.

Knowing the distribution parameters allow direct expression of the relationship between the need for spare parts and actual time of product operation, which is a general solution, that can appreciate better the product's behavior over time.

The failure intensity model was applied through a case study, to establish the necessary spare parts for three of the components of a loading, transportation and storage machine, namely the linear hydraulic engine for bucket handling, hydraulic piston pump from the braking circuit and brake pads from the composition of the same system, of which reliability was being analyzed.

For the two methods of calculation, the results are comparable, the differences being small especially for high confidence levels.

The study conducted shows that the number of spare parts required for a period of four years is high, demonstrating the low level of reliability of the analyzed products.

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